Automatic generation of mold-piece regions and parting curves for complex CAD models in multi-piece mold design

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Multi-piece molding technology is an important tool in producing complex-shaped parts that cannot be made by traditional two-piece molds. However, designing multi-piece molds is also a time-consuming task. This paper proposes an approach for automatic recognition of mold-piece regions and parting curves for free-form CAD models. Based on the geometric properties of objects and mathematical conditions of moldability, a collection of feasible parting directions is formed from which the sets of visible-moldable surfaces are identified for each parting direction. Moldable surfaces that are partially visible to a particular parting direction are recognized and divided into fragments by silhouette detection and edge extrusion. A set of criteria is proposed to arrange tentative fragments, which can be simultaneously visible to several parting directions, into appropriate regions for mold pieces. Finally, the parting curves for mold pieces are extracted from the corresponding mold-piece regions. The proposed algorithm overcomes the problems found in previous multi-piece molds and at the same time achieves high accuracy and high performance. Examples of industrially complex models are used to demonstrate the performance and robustness of the proposed algorithm. The approach is generic in nature, allowing its application to be extended to any complex geometry in 3-D mold design.

1. Introduction

Injection molding is one of the most common manufacturing processes extensively used today; it is able to produce parts with good quality and accuracy. From a geometric perspective, molds can be either two-piece or multi-piece. Conventional two-piece molds have only one primary parting direction, which constrains the two mold pieces to one axis of motion. However, undercuts can often be found in complex parts and, as a result, a number of side-cores is required to form the shapes of these undercuts. The higher the number of side-cores required, the higher the tooling costs and time consumed in the machine. Some complex parts with multiple undercuts may even be impossible to produce with two-piece molds. On the contrary, the use of multi-piece molds can overcome the aforementioned restrictions of traditional two-piece molds. With many different parting directions allowing free movement of mold pieces, undercuts can be eliminated so that no actual side-cores are required, thus significantly reducing total manufacturing costs. Consequently, multi-piece molding has become an important technology in handling complex geometries that cannot be made solely by using two-piece molds. Fig. 1 shows examples of typical molded parts, two-piece molds, and multi-piece molds provided by Protoform GmbH.

The main challenge in multi-piece mold design lies in identifying feasible parting directions, mold-piece regions, and the corresponding parting curves and parting surfaces. In this paper, a systematic approach to automating the determination of these requisite data for constructing mold pieces is proposed. The basic problem of multi-piece molds can be described thus: given a free-form CAD model, how do we determine the feasible parting directions \( D \), mold-piece regions \( S \), for each element \( d_i \) of \( D \), and the corresponding parting curves?

The remainder of the paper has been organized in the following manner: in Section 2, related works are reviewed and an overview of the proposed algorithm is presented. Background information of surface moldability and ray testing for surface visibility are described in Section 3. Section 4 discusses the algorithm for automatic recognition of mold-piece regions and parting curves. Several proposed criteria are also included. The algorithm implementation and examples are discussed in Section 5. Conclusions are given in Section 6.
2. Related works and overview of the algorithm

2.1. Related works

Automated processes for mold design have been developed in many publications. Most work focuses on traditional two-piece molds, including the determination of parting direction, undercuts, parting curves, and parting surfaces. Fu et al. [1] introduced an algorithm related to the determination, classification, and recognition of feature parameters for detecting undercuts. Ismail et al. [2] proposed a method to recognize cylindrical-based features based on an edge-boundary classification technique. Kharderkar et al. [3] described an algorithm to identify and display undercut features by implementing the Gauss map. Building upon these related techniques, Fu et al. [4] proposed an algorithm that determined the optimal parting direction in injection-molded parts by the number of undercut features and their corresponding volumes. Furthermore, Chen et al. [5] posed a method in which three possible parting directions were defined by surface normal vectors of a bounding box. Feasible parting directions were then estimated based on the dexter model and fuzzy decision-making.

In order to determine parting curves and parting surfaces, Fu et al. [6] described a technique employing the maximum external edge loops between the core- and cavity-molded surface groups. Chakraborty et al. [7] presented a method to determine the parting curve and parting surface for a two-piece permanent mold based on a combination of the surface area of the undercut, the flatness of the parting surface, and the draw depth. In addition, Wong et al. [8] proposed an uneven slicing approach to finding the feasible parting curves of a CAD model. The optimal parting curve was evaluated based on the criteria described by Ravi and Srinivasan [9]. Furthermore, Paramio et al. [10] evaluated the deformability of injection-molded parts through the slicing of their CAD models, which could then be used to identify feasible parting curves. For the generation of parting surfaces, Li et al. [11,12] posed an approach by evaluating the extrudability of parting curves; a subdivision technique was employed to generate parting surface regions for the portions of the parting curves that were not extrudable.

Side cores or pins can be generated along with the recognition of undercuts. Banerjee et al. [13] used the retraction space of each undercut surface to identify the shapes of the side cores. The undercut surfaces were grouped into undercut regions according to a discrete set of feasible and non-dominated retractions, after which the geometry of individual side cores could be obtained. Fu [14] used the concepts of surface visibility, deformability, and moldability to identify the surfaces molded by side cores. Furthermore, Ran and Fu [15] described an algorithm for automatic design of internal pins after identifying the inner undercuts and extracting the related surfaces.

The development of CAD research for mold design has also been connected to manufacturability and manufacturing costs. Bidkar et al. [16] presented a feature recognition method based on the elemental cubes to assess the critical manufacturability information of injection-molded parts. Denkova et al. [17] introduced a method in which a CAD-based application of the calculation tool ‘visual form calculator’ was used to generate and analyze CAD models of mold cavities in order to compute tool accessibility and manufacturing costs.

In the area of multi-piece molding, few articles have been published. Dhaliwal et al. [18] presented a feature-based approach to automatic design of multi-piece sacrificial molds. In their approach, the desired gross mold shape is decomposed into simpler shapes for manufacturability and assemblability purposes. By the same rationale, Huang et al. [19] described an algorithm for generating multi-piece sacrificial molds with an accessibility driven spatial partitioning scheme. Chen and Rosen [20,21] introduced a region-based method for partitioning parting surfaces into regions and combining them into mold pieces. The basic elements in their approach are concave regions and convex surfaces. A reverse glue operation is then proposed to automate the construction of multi-piece molds based on the generation of parting surfaces. In addition, Priyadarshi et al. [22] described a geometric algorithm for automatic design of multi-piece permanent molds. The mold pieces are constructed based on the results of a global accessibility analysis of the part.

2.2. Problems of multi-piece mold design

As summarized previously, the most significant advantage of multi-piece molds over conventional two-piece molds is that they can be used to handle complex-shaped parts. However, current approaches to multi-piece mold design have two limitations. First, they require simple polyhedral parts or approximate complex parts by facets [18–22], which may not be acceptable in the industry. Second, some algorithms have limited application domains. For instance, the algorithms proposed by Dhaliwal [18] and Huang [19] do not handle general partitioning cases such as partitioning along non-planar faces. Moreover, in Huang’s approach, manufacturability is measured only by the number of cuts involved in the partitioning. In reality, manufacturability is more directly related to the number of components and their geometric complexity. In Chen and Rosen’s method [20,21], only local disassemblability evaluation is performed when generating parting surfaces. As a result, the current paper proposes a systematic approach to automatic recognition of mold-piece regions and parting curves for constructing mold pieces. In the proposed approach, several criteria and techniques, such as visibility-ray testing and silhouette detection, are developed to determine moldable surfaces and appropriate surface regions for mold pieces. The algorithm is sufficiently generic to be applied in commercial CAD systems.

2.3. Overview of the proposed algorithm

All curved/free-form surfaces of the input CAD model are inserted into the proposed system. Information of the geometric entities of the model (vertices, edges, and surfaces) is also extracted and used as input for the algorithm. Equations describing edges and surfaces are formed based on such information. The following steps have been developed to generate suitable parting directions and parting curves for the piece.
The physical meaning of Eq. (1) is that rays from infinity that are parallel to the parting direction $d_i$ of the collection $D$ are used to determine the different regions for mold pieces. In the case where a surface simultaneously belongs to two or more surface sets, the surface is rearranged into the most appropriate mold-piece region. This is described in Section 4.4.

(4) All visible-moldable surfaces of sets $S_i$ are used to determine the different regions for mold pieces. In the case where a surface simultaneously belongs to two or more surface sets, the surface is rearranged into the most appropriate mold-piece region. This is described in Section 4.4.

(5) Finally, the algorithm for locating parting curves of mold-piece regions is implemented. All outer and inner loops of the parting curves are determined for further generation of mold-piece parting surfaces and structures. This process is detailed in Section 4.5.

3. Background information on moldable surfaces and ray testing for visibility

In order to describe the proposed algorithm, the definitions of moldable surfaces, which are extended from those of surface visibility and moldability mentioned in [6], are first presented. This section focuses on the following three types of surfaces: planar surfaces (first-order surfaces), quadric surfaces, and free-form surfaces (third-order surfaces or higher). It is assumed that $s_i$ is a surface of model $M$ and $n_i$ is the normal vector of an arbitrary point on $s_i$. Let $d_i$ be one of the parting directions of model $M$. Surface $s_i$ is moldable in direction $d_i$ if the following condition is met:

$$n_i \cdot d_i \geq 0.$$  

The physical meaning of Eq. (1) is that rays from infinity that are parallel to the parting direction $d_i$ cannot cast any shadow in this direction onto the surface; in other words, the surface is visible in these rays. Equality occurs when the surface is a vertical wall.

For planar and quadric surfaces, the normal vectors of points on these two types of surfaces possess exact directions, thus allowing the fulfillment of Eq. (1) to be easily confirmed (refer to Fig. 2(a) and (b)). However, for a free-form surface, a sub-division method must be employed: the surface is divided into small regions with equal distances in parametric coordinates $u$ and $v$, and the normal vector $n_{ij}$ at a corresponding node $(i,j)$ of the surface is determined. The entire set of normal vectors is then used to confirm the fulfillment of Eq. (1). Only if the equation is met for all nodes can the surface be defined as moldable. Fig. 2(c) shows an example of a moldable free-form surface along parting direction $d_i$.

For surfaces of revolution, a substantially different method must be applied to determine the moldable surfaces: the axis of revolution is used, instead of the normal vector, to ascertain whether a revolved surface can be molded. In general, cylinders and cones are present in the design of industrial parts. When the surface is a cylinder, the condition for moldability is

$$a_s \cdot d_i = 1$$

where $a_s$ is the axis of the cylinder. Eq. (2) asserts that a cylindrical surface is moldable if its axial vector is parallel to the parting direction $d_i$. As for conical surfaces, the following equation is used:

$$a_s \cdot d_i \geq \cos \left( \frac{\alpha}{2} \right)$$

where $a_s$ is the axis of the cone and $\alpha$ is the angle at the apex. Eq. (3) shows that a conical surface is moldable if the angle between its axis and parting direction $d_i$ is smaller than $\alpha/2$. Moreover, this situation allows removal of the part from the mold without any difficulty caused by these conical surfaces. Fig. 3 shows an example of moldable and unmoldable conical surfaces.

Once a surface is defined as moldable in a certain direction, its visibility must be examined. Accordingly, all moldable surfaces in each parting direction $d_i$ are collected and stored in a so-called ‘$S$-table’. Each moldable surface $s_i$ is then tested in sequence by rays to determine whether it is obscured by other surfaces included in the $S$-table. Each ray is originated from the end point or the middle point $p_i$ of each edge $e_i$ of the test surface $s_i$ along direction $d_i$. If the rays do not intersect with other surfaces in the $S$-table except the points $p_i$ themselves, the test surface $s_i$ is identified as a visible one; otherwise, $s_i$ is identified as an invisible surface. Examples of visible and invisible moldable surfaces in a direction $d_i$ of a model are shown in Fig. 4.

4. Algorithm for automatic parting curve generation of multi-piece molds

4.1. Collection of tentative parting directions

In multi-piece mold design, the parting direction is the direction along which a mold piece can be separated without any obstacles, and thus the determination of feasible parting directions is a priority issue that needs to be addressed. In our approach, a collection $D$ of tentative parting directions is formed based on the geometrical properties of the part’s features by considering the following three types of directions: relative coordinate axes, axes of features of revolution, and normal directions of planar surfaces (which follows...
that of [22]). These are the directions from which most surfaces of the part can be accessed. Fig. 5 shows a typical part to be analyzed during the design process of multi-piece molds. All tentative directions for accessing the surfaces of the part are found using the properties of its geometric features; these are presented with different colored arrows in the figure. Note that for each tentative parting direction \( \mathbf{d}_i \) in \( D, -\mathbf{d}_i \) is also included in the collection. The part shown in the figure will be used as a running example for describing steps of the proposed algorithm.

### 4.2. Formation of visible-moldable surfaces

The accessibility of all surfaces of a part is analyzed according to the tentative parting directions in collection \( D \). Visible-moldable surfaces for each direction \( \mathbf{d}_i \) in \( D \) can then be found. The process is performed in the following steps:

**Step 1:** Identify moldable surfaces for direction \( \mathbf{d}_i \) based on Eqs. (1)-(3). All moldable surfaces related to each parting direction \( \mathbf{d}_i \) are grouped into a set \( S_i \).

**Step 2:** For each moldable surfaces set \( S_i \), a ray test, as presented in Section 3, is performed to determine whether this surface is visible in direction \(-\mathbf{d}_i\). Invisible surfaces, which are obscured by other surfaces in \( S_i \), are removed from \( S_i \).

As shown in Fig. 6, both the red and blue surfaces of the part are identified as moldable surfaces along parting direction \( \mathbf{d}_i \). However, the red surface is completely obscured by other surfaces when viewed from infinity along direction \(-\mathbf{d}_i\). Thus, the red surface is removed from visible-moldable surface set \( S_i \).

During the process of identifying visible-moldable surfaces, there may be two cases where the surfaces are partially visible. The first is called the 'dual moldable surface' where Eqs. (1), (2), and/or (3) are satisfied only by a group of points on these surfaces (see the green surfaces in Fig. 7(a)). The second is the 'partially obscured surface' in which only a part of its area is obscured by other surfaces (see the red surface in Fig. 7(b), which is partially obscured by the blue surfaces when viewed from infinity along \(-\mathbf{d}_i\).

#### (a) Dual moldable surface

A silhouette-detecting algorithm is employed to identify the exact boundary between the visible and invisible regions of a dual moldable surface. First, a dual moldable surface \( S(u, v) \) is described by a NURBS equation, which is generally in the following form:

\[
S(u, v) = \begin{pmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{pmatrix} = \frac{\sum_{i=0}^{n_1} \sum_{j=0}^{n_2} B_{i,k}(u) \cdot B_{j,l}(v) \cdot w_{i,j} \cdot C_{i,j}}{\sum_{i=0}^{n_1} \sum_{j=0}^{n_2} B_{i,k}(u) \cdot B_{j,l}(v) \cdot w_{i,j}} \quad w_{i,j} > 0
\]

(4)

where surface \( S(u, v) \) has degree \( k \) in parameter \( u \) and degree \( l \) in parameter \( v \), \( C_{ij} \) is the control point, \( w_{ij} \) is its corresponding weight, \( n_1 \) and \( n_2 \) are the number of control points in directions \( u \) and \( v \), and \( B_{ik}(u) \) and \( B_{lj}(v) \) are the \( B \)-spline basis functions defined in \( u \) and \( v \). The silhouette of a free-form object is typically defined as the set of points on the object's surface where the surface normal vector is perpendicular to the vector from the viewpoint. A point on surface \( S(u, v) \) with corresponding surface normal vector \( N(u, v) \) is a silhouette point if the angle between the parting direction \( \mathbf{d}_i \) and \( N(u, v) \) is \( 90^\circ \). This means that the following constraint must be met:

\[
\mathbf{d}_i \cdot N(u, v) = 0
\]

(5)

in which \( N(u, v) \) is computed by the following rational equation:

\[
N(u, v) = \begin{pmatrix} N_{x}(u, v) \\ N_{y}(u, v) \\ N_{z}(u, v) \end{pmatrix} = \frac{\partial S(u, v)}{\partial u} \times \frac{\partial S(u, v)}{\partial v}.
\]

(6)

Let \( Nm(u, v) = \sum_{i=0}^{n_1} \sum_{j=0}^{n_2} B_{i,k}(u) \cdot B_{j,l}(v) \cdot w_{ij} \cdot C_{ij} \); its partial derivative with respect to parameter \( u \) is \( Nm_u(u, v) = \sum_{i=0}^{n_1} \sum_{j=0}^{n_2} B_{i,k}(u) \cdot B_{j,l}(v) \cdot w_{ij} \cdot C_{ij} \). Also, let \( Dn(u, v) = \sum_{i=0}^{n_1} \sum_{j=0}^{n_2} B_{i,k}(u) \cdot B_{j,l}(v) \cdot w_{ij} \) so that its partial derivative with respect to parameter \( u \)
is \( Dn_u(u, v) = \sum_{i=0}^{n_u} \sum_{j=0}^{n_v} B^u_{i, j}(u) \cdot B^v_j(v) \cdot w_{i,j} \cdot B^v_{i, j} \) is the derivative of a \( B \)-spline basis function and can be expressed by the following equation:

\[
B^u_{i, k}(u) = k \frac{1}{u_{i+k} - u_i} \frac{B_{i,k-1}(u)}{u_{i+k-1} - u_{i+1}} + \frac{k-1}{u_{i+k-1} - u_{i+1}} B_{i+1,k-1}(u).  \tag{7}
\]

Therefore, the partial derivative of \( S(u, v) \) with respect to \( u \) is:

\[
\frac{\partial S(u, v)}{\partial u} = \left( \frac{Nm(u, v) \cdot Dn(u, v) - Dn_u(u, v) \cdot Nm(u, v)}{Dn^2(u, v)} \right) = \frac{Nm'_u(u, v) \cdot Dn(u, v) - Dn'_u(u, v) \cdot Dn(u, v) \cdot S(u, v)}{Dn^2(u, v)}
\]

\[
= \frac{Nm'_u(u, v) \cdot Dn'_u(u, v) \cdot S(u, v)}{Dn(u, v)}.
\tag{8}
\]

Similarly, the partial derivative of \( S(u, v) \) with respect to \( v \) is:

\[
\frac{\partial S(u, v)}{\partial v} = \left( \frac{Nm'_u(u, v) \cdot Dn'_u(u, v) \cdot S(u, v)}{Dn(u, v)} \right).
\tag{9}
\]

A silhouette curve can be found by substituting Eqs. (6)–(9) into Eq. (5). In fact, Eq. (5) is a polynomial equation of two variables \( u \) and \( v \). In this scenario, there are fewer equality constraints than there are variables. Its zero-solution set can be computed based on the convex hull and subdivision properties of rational spline functions. A set of discrete points approximating the zero set is generated by recursive subdivision based on the Newton–Raphson method. The method to solve a general system of \( m \) polynomial equations of \( n \) variables is described by G. Elber and M.S. Kim [23]. Fig. 8 shows an example of surface regions divided from dual moldable surfaces of the pedal part in Fig. 5.

(b) Partially obscured surface

This type of surface will be divided into regions that are completely visible or invisible to the current parting direction through an edge-extrusion algorithm, as illustrated in Fig. 9. In the figure, surface \( s_1 \) is partially obscured by surface \( s_2 \). Each boundary edge of \( s_2 \) is used to build an extrusion surface \( s_3 \) along the current parting direction. The intersection between surface \( s_1 \) and surface \( s_3 \) can then be used to divide \( s_1 \) into an un-obscured region and an obscured region.

Fig. 10 shows an example of the surface regions divided from a partially obscured surface of the part in Fig. 5. The red surface is partially obscured by others when viewed from infinity along directions \( -d_3 \), \( -d_5 \), and \( -d_6 \). The edge-extrusion algorithm is thus applied, resulting in the eleven surface regions divided from the red surface as shown.

4.3 Determination of additional parting directions

Assuming that there are some ‘inaccessible’ surfaces invisible from the directions in \( D \), an analysis is performed to identify additional parting directions (APDs) along which such surfaces are visible. APDs found are inserted into the collection \( D \).

The determination of APDs is implemented via the use of \( G \)-map and \( V \)-map introduced in [24]. A \( G \)-map is a map of surface normals onto a unit sphere, where each point on the map represents the intersection of the transferred surface normal vector with the surface of the unit sphere. On the other hand, a \( V \)-map of a surface is a set of points in the unit sphere whereby every point in the \( V \)-map deviates from the corresponding point in the \( G \)-map by an angle less...
than 90°. To identify the $G$-map and $V$-map of each free-form surface of the CAD part, we employ Eqs. (6)–(9) in computing the normal vectors $\mathbf{n}_{ij}$ at the corresponding nodes $(i, j)$ of each surface.

Fig. 11 illustrates the determination of an APD using a $V$-map. The pocket feature of the example part in Fig. 11(a) has five inaccessible surfaces, each of whose $V$-maps is a hemisphere (Fig. 11(b)). The intersection of these hemispherical $V$-maps forms the $V$-map of the pocket. The vector passing through the central point of the $V$-map and its origin located at the unit sphere is the APD of the pocket (Fig. 11(c)).

In the proposed algorithm, each of APDs is only identified for inaccessible surfaces which are in the same undercut feature. Of course, if the determination process of an APD is applied to inaccessible surfaces belonging to different undercut features, it is possible that no feasible parting direction can be found. Therefore, all inaccessible surfaces must be classified into different groups before the $V$-map and $G$-map methods can be used. Each group consists of adjacent inaccessible surfaces connected together to form an undercut feature (Fig. 12).

To classify inaccessible surfaces into different groups, all inaccessible surfaces are collected and numbered from $s_1$ to $s_n$ (where: $n$ is the total number of inaccessible surfaces). Each of inaccessible surface $s_i$ will be examined to find its adjacent surfaces. In the process, if two surfaces have at least one common edge, they are considered as adjacent ones. Once inaccessible surface $s_j$ is determined as the adjacent surface of $s_i$, $s_j$ will be inserted into the same group of $s_i$, or in other words, they are in the same undercut feature.

Importantly, an APD may be a possible parting direction for the visible-moldable surface set $S_i$ identified previously. As shown in Fig. 13, groups $s_1$, $s_2$, and $s_3$ are identified as visible-moldable surfaces associated with parting direction $d_1$ while all surfaces of the pocket are visible-moldable with the APD $d_7$ determined by the $V$-maps. However, with $d_2$, the surface group $(s_1, s_2, s_3)$ and the surfaces of the pocket are all visible-moldable. In such a case, the surfaces are re-classified into the same set associated with APD $d_7$. Therefore, once an APD of an undercut feature is identified,
its accessibility to the surfaces of each set $S_i$ must be analyzed. If all surfaces of $S_i$ are accessible by the APD, the surfaces of $S_i$ are grouped with the surfaces of the undercut feature to create a new surface set associated with the APD. This process ensures that the number of mold pieces is reduced to a minimum.

4.4. Formation of regions for mold pieces

The regions for mold pieces are determined using all visible-moldable surfaces of each set $S_i$ associated with direction $d_i$. These regions are groups of adjacent surfaces connected together to form a united region. In the proposed algorithm, a region of a mold piece is called ‘mold-piece region’, and is denoted by $R_i$. Individual visible-moldable surfaces and surface regions divided from dual moldable surfaces or divided from partially obscured surfaces are generally called ‘fragments’. In reality, a fragment of surface sets $S_i$ may either belong to exactly one mold-piece region associated with one parting direction $d_i$ (in which case it is an ‘exact fragment’, denoted by $f^e$), or to several mold-piece regions associated with several parting directions (in which case it is a ‘tentative fragment’, denoted by $f^t$). Rearrangement of tentative fragments in surface sets $S_i$ into appropriate regions is the most important process in the formation of mold-piece regions. Three criteria are proposed for this work.

**Criterion 1.** If a tentative fragment $f^t$ is passed through when withdrawing the mold piece of an exact fragment $f^e$ along its corresponding parting direction, the fragment $f^t$ is rearranged to the same mold piece region as the fragment $f^e$.

As shown in Fig. 14, the fragment $f^t$ is obscured by other surfaces when viewed from infinity along direction $-d_3$. Thus, it is an exact fragment that only belongs to the mold piece associated with parting direction $d_3$. Conversely, $f^t$ is a tentative fragment that is visible in both directions $-d_3$ and $-d_5$; hence, it can belong to either mold pieces associated with $d_3$ or $d_5$. However, the mold piece containing $f^e$ will pass through $f^t$ if it is withdrawn along direction $d_3$. Therefore, $f^t$ is rearranged into the same region as $f^e$.

To check whether a tentative fragment $f^t$ is passed through by the mold piece of the exact fragment $f^e$, we create extrusion surfaces from edges of $f^e$ along the parting direction. If the interior of $f^t$ intersects with the extrusion surfaces, $f^t$ is considered to be passed through by the mold piece of $f^e$.

**Criterion 2.** When a tentative fragment $f^t$ is adjacent to several exact fragments $f^e$ of several mold-piece regions, the tentative fragment is rearranged into the mold-piece region having the most exact fragments adjacent to $f^t$.

As shown in Fig. 15, fragments $f^e_1$ and $f^e_2$ are exact fragments of the mold-piece region associated with parting direction $d_5$. Fragment $f^t_1$ is a tentative fragment adjacent to both $f^e_1$ and $f^e_2$. Moreover, there are no exact fragments of other mold-piece regions adjacent to $f^t_1$. Thus, $f^t_1$ is rearranged into the same mold-piece region as $f^e_1$ and $f^e_2$.

**Criterion 3.** A tentative fragment $f^t$ is rearranged into the mold-piece region associated with the parting direction along which the withdraw distance of $f^t$ is minimal.

The withdraw distance of a tentative fragment $f^t$ is measured by the distance along the parting direction from the farthest point of $f^t$ to the surface of the bounding box of the molded part. Fig. 16 shows two withdraw distances of a tentative fragment; these correspond to two parting directions $d_3$ and $d_5$, and are measured from the farthest points $p_2$ and $p_1$, respectively.

In Fig. 17, the tentative fragment $f^t$ can belong to either of the mold-piece regions associated with parting directions $d_3$ and $d_5$. However, the withdraw distance of $f^t$ corresponding to direction
\( d_3 \) is less than that corresponding to \( d_5 \). Therefore, \( f^t \) is rearranged into the mold-piece region associated with direction \( d_3 \).

The order of applying the three criteria must be determined to rearrange tentative fragments. Among the three criteria, \textit{Criterion 1} must be applied first. It is used until no more tentative fragment passed through by the mold-pieces of the exact fragments has been found. That is because if a tentative fragment \( f^t \) is rearranged into a mold-piece region based without first checking \textit{Criterion 1}, the corresponding mold-piece region created may not valid as a united region. As shown in Fig. 18, the tentative fragment \( f^t_1 \) is passed through by the exact fragment \( f^e_1 \) when withdrawing the mold-piece of \( f^e_1 \) along the direction \( d_5 \). If \( f^t_1 \) is rearranged into other mold-piece region different from that of \( f^e_1 \) (according to the \textit{Criterion 2} or \textit{3}), the mold-piece region of \( f^t_1 \) will have a gap in between and it cannot create a valid mold-piece structure.

Next, \textit{Criterion 2} should be applied before \textit{Criterion 3}. In reality, if a tentative fragment is processed by first considering \textit{Criterion 3}, it can create a very complicated or an invalid mold-piece structure. As shown in Fig. 19, tentative fragments \( f^t_4 \sim f^t_8 \) are only adjacent to exact fragments \( f^e_4 \sim f^e_8 \) of the mold-piece region \( R^3 \) associated with the direction \( d_3 \). Conversely, the withdraw distances of \( f^t_4 \sim f^t_8 \) along the direction \( d_3 \) are shorter than that of \( f^e_4 \sim f^e_8 \) along the direction \( d_5 \). If \( f^t_4 \sim f^t_8 \) are rearranged into the mold-piece region \( R^3 \) due to the consideration of \textit{Criterion 3}, both the mold-piece region \( R^3 \) and \( R^5 \) cannot create reasonable mold-piece structures which cause difficulties in industrial machining.

Moreover, the mold-piece created from \( R^3 \) may not be withdrawn along the direction \( d_3 \).

Besides the proposed criteria, the order in which the sets \( S_i \) of visible-moldable surfaces are processed in computing the corresponding mold-piece regions affects the whole geometries of multi-piece molds. As mentioned in Section 4.1, parting directions are selected from a set of different tentative directions. Therefore, the sets \( S_i \) are divided into three groups: a group of \( S_i \) associated with coordinate axes, a group associated with axes of revolution features and normal directions of planar surfaces, and a group associated with \textit{APDs}. In the proposed algorithm, the following two directives are utilized for these groups: (1) The group associated with the coordinate axes is processed before the group associated with axes of revolution features or normal directions of planar surfaces. The group associated with \textit{APDs} is processed last. (2) For
In each group, the processing orders are based on the total area of their included surfaces. The set $S_i$ with the largest total surface area is processed first and the one with the smallest is processed last. After these directives, all sets $S_i$ are rank ordered and indexed from $i = 1$ to $i = i_{max}$ (where $i_{max}$ is equal to the total number of sets $S_i$).

All tentative fragments are then rearranged into appropriate mold-piece regions $R^i$ with the processed order of sets $S_i$ and the criteria proposed above. The complete mold-piece region algorithm (MPR algorithm) is presented in Fig. 20. After the MPR algorithm is performed, mold-piece regions associated with all parting directions $d_i$ are identified to further generate parting curves and construct mold-piece structures.

The application of the MPR algorithm is illustrated for all fragments shown in Fig. 21. In the figure, $f_1^i$, $f_2^i$, $f_3^i$, and $f_4^i$ are exact fragments of $S_5$, and can only belong to the mold-piece region associated with parting direction $d_5$; $f_5^i$ is an exact fragment of $S_3$, and can only belong to the mold-piece region associated with parting direction $d_3$. In processing set $S_5$, the tentative fragment $f_1^i$ is adjacent to $f_2^i$ and is passed through by the mold piece of $f_1^i$ associated with direction $d_5$. Thus, $f_1^i$ is rearranged into the same mold-piece region as $f_2^i$ (following Criterion 1). Fragment $f_1^i$ is immediately updated to be a new exact fragment for further processing. Next, fragment $f_3^i$ is adjacent only to exact fragments of the mold-piece region associated with $d_5$; it is therefore rearranged into this mold-piece region (adhering to Criterion 2). Similarly, $f_1^i$ is immediately updated to be a new exact fragment. Criterion 2 is also repeated for tentative fragments $f_4^i$ through $f_8^i$.

In processing $S_3$, tentative fragments $f_2^i$ and $f_11^i$ are rearranged into the same mold-piece region as the exact fragment $f_3^i$ associated with parting direction $d_3$ (again following Criterion 1). Meanwhile, $f_9^i$ and $f_{10}^i$ are tentative fragments that can belong to both mold-piece regions associated with $d_3$ and $d_5$. However, the withdraw distance of $f_{10}^i$ and $f_{10}^i$ corresponding to $d_3$ is less than that corresponding to $d_5$. Hence, $f_{10}^i$ and $f_{10}^i$ are rearranged into the mold-piece region associated with direction $d_3$ (as stipulated by Criterion 3). This analysis continues for other fragments until tentative fragments are exhausted.

4.5. Location of parting curves for mold pieces

Finally, all mold-piece regions $R^i$ are used to obtain parting curves for mold pieces. In general, a parting curve is a closed loop that identifies surfaces at which the mold is split into different pieces. Hence, parting curves are the boundaries of mold-piece regions. There may exist one or several closed loops depending on the existence of ‘hole-features’. If there is more than one closed loop, the following technique is employed to identify the outer and inner loops: all loops are projected onto a plane perpendicular to the parting direction associated with the mold-piece region under

**Fig. 20.** Flow diagram of generation of mold-piece regions.

**Fig. 21.** Fragments for applying the MPR algorithm.
The process for determining the boundary of each mold-piece region involves the removal of all common edges. First, all edges of the surfaces of a mold-piece region are checked to identify common edges. In particular, an edge $e$ is determined to be a common edge ($e_{CM}$) of two adjacent surfaces if the following equation is met:

$$ e \in (CB(s_j) \land CB(s_k)) \quad j, k = 1, 2, 3 \ldots n $$

where $s_j$ and $s_k$ are surfaces of the mold-piece region whose parting curve is to be determined, $n$ is the total number of surfaces in this region, and $CB(s_j)$ and $CB(s_k)$ are the corresponding closed boundary loops of surfaces $s_j$ and $s_k$, respectively. In Eq. (10), the term $(CB(s_j) \land CB(s_k))$ refers to the edges simultaneously belonging to the closed boundary loops of both surfaces $s_j$ and $s_k$. Now, let $\Sigma_1$ be the set of all edges of all surfaces in the mold-piece region and $\Sigma_2$ be the set of all common edges $e_{CM}$; then the parting curve $e_{pc}$ of a mold-piece region is determined as follows:

$$ e_{pc} = (\Sigma_1 - \Sigma_2). $$

The physical meaning of Eq. (11) is that the parting curves of a mold-piece region are the remaining external edges after subtracting all the common edges between adjacent surfaces included in the region. Fig. 22(a) shows a mold-piece region associated with one parting direction of the molded part mentioned in Figs. 5, and 22(b) is its parting curve. As seen from the figure, the boundary edges form one outer loop and three inner loops from which the parting surfaces and mold piece may be suitably constructed.

5. Implementation

The proposed algorithm has been implemented using the API-based Pro/Toolkit for Pro/Engineer Wildfire 5.0. Geometric information of the CAD parts (vertices, edges, and surfaces) is extracted and used as input for the algorithm. The part featured in this research is a pedestal; it has been used as a running example in the previous sections to illustrate our algorithm. The proposed system analyzed all surfaces of the part, identifying the feasible parting directions and corresponding visible-moldable surfaces. Six directions $d_1 \sim d_6$ were found relative to the coordinate axes; these formed a sufficient collection of parting directions. By using the MPR algorithm discussed in Section 4.4, six mold-piece regions were determined based on the surface sets’ visibility and moldability, as shown in Fig. 23. The final parting curves for each mold piece were generated by connecting the external edges of outer surfaces after subtracting all common edges. As seen from the figure, these parting curves can be used directly to generate parting surfaces for mold design.

To demonstrate in more details the ability of the proposed system, another complex CAD part is used. The part is a plug model. As shown in Fig. 24, six possible directions denoted from $d_1$ to $d_6$ are found to form a sufficient collection of parting directions.

When determining visible-moldable surfaces sets for each parting direction $d_i$, there are some tentative fragments which can belong to several mold-piece regions (Fig. 25). They are actually fragments divided from partially obscured surfaces when viewed from infinity along the direction $-d_3$. Thus, the fragments from $f_{21}^{t}$ to $f_{26}^{t}$ are rearranged into the same mold-piece region $R^5$ with $f_{25}$ (following to Criterion 1). It is immediately updated to be a new exact fragment for further process. It is similar to other tentative fragments $f_{21}^{t}, f_{27}^{t}$. Besides, tentative fragments $f_{25}, f_{26}$ are adjacent to most of exact fragments of $S_5$, therefore they are also rearranged into mold-piece region $R^2$ (adhering to Criterion 2). Meanwhile, $f_{25}^{t}$ is the tentative fragment...
that can belong to several mold-piece regions associated with $d_1$, $d_2$, $d_3$, and $d_4$. However, the withdraw distance of $f_{28}$ along to $d_5$ is shortest. Hence, $f_{28}$ is rearranged into the mold-piece region $R^5$ associated with direction $d_5$ (as stipulated by Criterion 3). The analysis continues for other fragments of $S_5$ until no more tentative fragment is found.

The argument is similar for tentative fragments $f_{11}$, $f_{12}$, and $f_{13}$ of surfaces set $S_1$. After the implementation of the MPR algorithm, all tentative fragments are rearranged into appropriate mold-piece regions associated with parting directions $d_1 \sim d_6$. Fig. 26 shows six mold-piece regions and their corresponding parting curves for the example part. They are then used to extract successfully six mold-pieces as shown in Fig. 27.

For efficient implementation of the algorithm, a filtered program was employed to prune unnecessary information. After the CAD model was loaded, the working mode of the system did not require any adjustment from the user. Through the running example—the pedal part that we used to present the capacity of the algorithms, the computation time was acceptable although it was not a simple part. It found all tentative parting directions of the set $D$ (following the rules described in Section 4.1) in about two seconds. Based on the three proposed criteria, the MPR
algorithm successfully created six valid mold-piece regions and corresponding parting curves from the sets of visible-moldable surfaces within 5 min. For the plug part, the total computation time is about 4 min and 12 s. On other complex parts it is capable of finding feasible solution. However, it takes the computation time more than 5 min. The reason is that the optimality may not be guaranteed in our programming. This will be updated in the future work.
The numerical experiments show that for high-order interpolation (fourth order or higher) it can be more convenient to make distributions of nodes of curves/surfaces to reduce the condition numbers of the resulting elemental matrices. Moreover, using a good distribution of nodes, the number of computational matrices can be reduced two or three times, in comparison with the original distribution of nodes.

6. Limitations

There are some limitations and future works can be addressed as follows. First, the set $D$ of tentative parting directions is determined based on three types of directions: relative coordinate axes, axes of features of revolution, and normal directions of planar surfaces. These directions are always good candidates for a parting direction of a mold-piece. If there are surfaces that cannot be accessible from any of the directions in $D$, the APD’s algorithm will be applied to detect a feasible direction for these inaccessible surfaces. However, it may be possible that even after applying the APD’s algorithm, these surfaces are still inaccessible as shown in Fig. 28. The pocket $A$ is located inside the pocket $B$. The APD (denoted by the blue vector) of inaccessible surfaces of the pocket $A$ intersect with a surface of the pocket $B$. Therefore, the corresponding mold-piece created from inaccessible surfaces of the pocket $A$ cannot be freely withdrawn along only the APD direction. In the proposed algorithm, such part is rejected as non-moldable. Although for industrial parts, such cases are rare, the improvements for the proposed algorithm to create split-cores for such parts must be required in the future works.

Second, using the visibility map to identify the APD direction of undercut features may not be satisfied in some cases. As shown in Fig. 29, the V-map of each planar surface is a hemisphere but the V-map of the entire part is empty. Indeed, there is not a feasible direction from which the interior of the model is visible from the exterior. The main reason is that the visibility map of an object is constructed using mainly the local accessibility information instead of the global accessibility. Although such kinds of structures are not common in industrial molded parts, the consideration of global accessibility is still required. Global accessibility also en-
the field of multi-piece mold design. There are two significant improvements with respect to the following characteristics:

- The approach can automatically recognize both mold-piece regions and parting curves for complex CAD models. Based on the three criteria and the MPR algorithm proposed in the paper, visible-moldable surface regions for each mold-piece are identified to generate reasonable mold-piece structures.
- Current approaches to multi-piece mold design normally require polyhedral parts or approximate complex parts by facets which may not be acceptable in industry. In the proposed approach, the system can handle surface models containing free-form surfaces by dividing surfaces into small fragments for rearranging into appropriate mold-piece regions. The capacity of the proposed system has been shown through examples described in the Implementation section.

Although there exist some limitations, we expect that the algorithms described in this paper will provide feasible foundations for automating the mold-piece regions and parting curves. Through illustrative example parts, the approach is proven robust and adaptable for use with current CAD/CAM systems and applications in industry. It will help in reducing the working time in the field of multi-piece mold design and manufacturing.

### References


