The Development of a Didactic Prototype for the Learning of Mathematics Through Augmented Reality

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Abstract

This work applies Augmented Reality technology in the educational process through a didactic prototype that promotes visualization skills related to the learning of mathematical content. An initial prototype has been designed and built with the purpose of arriving at 3 dimensional objects performing specific actions, in space and time, executed with 2 dimensional objects. The AR production of mathematical objects with which student may interact offers the opportunity to mentally record the process through which they are generated, favoring visualization skills. In the initial academic phase, an analysis of the first three college calculus courses was carried out. The objective was the identification of a transversal content suitable to be developed in AR environment. Once this content was established and discussed, the conceptualization of the prototype was carried out, identifying first the platform of technological and human resources available for the project. The technical phase was focused on developing the AR technology prototype around the didactic design concept. The adjustment decisions in this process were based around the academic-technical integration meetings. A pilot experience for exploratory purposes was developed with Mathematics I for engineering students during May 2013. The aim was to describe the actions the prototype encourages from the students and to capitalize these results to determine limitations and reaches of this first prototype, from a didactically and technically point of view. The pilot experience confirms that AR technology in education increases the current motivation to learn by students. The work aims to study about the development of didactic resources that serve students in the learning of a visual and tangible mathematics.

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1. Background

The work presented here is the outcome of collaboration between various academic branches in the Campus Monterrey, part of the Tecnológico de Monterrey, educational institution in Mexico. The guiding purpose of this collaboration is to establish emerging digital technologies as a central tool in the educational process. In particular this work refers to the learning process corresponding to Mathematics.

In said institution, a didactic method for teaching and learning is being practiced in Mathematics courses at a college level, promoting the transfer of Calculus knowledge to the different existing specialty branches. Through this method, the application of learned knowledge is emphasized, favoring the student’s understanding of the utility of this knowledge in the solving of real life problems [1, 8-9].

This different method of interaction with the content of the curricular sector of Calculus involves the use of technologies, including spreadsheets and specialized graphing software. These tools have been included in the development of mathematical knowledge through the didactic design, inviting maximization in learning efficiency by means of their use. This new discourse is explained in the three textbook collection: Applied Calculus: Mathematical Competencies through contexts [10-12].

Another subject worthy of mention is the record of technological production and innovation developed in the institution, which led to the collaboration of authors in the search of a new, favorable impact in education. Previous projects like the Augmented Reality engine available through Campus Monterrey (www.bienetec.es/) and their experience with products, services and technological solutions for 3D and AR, certify the feasibility of this new path product development, seeking a product that generates a synergy between educational, and technological knowledge [2, 5].

The project, Augmented Reality: a promoter of visualization in the learning of Calculus, is being developed in response to the Call for Innovation Novus 2012, an initiative of Tecnológico de Monterrey to favor projects in the field of educational innovation. With the project, a didactic prototype has been created, one that is explained in the present paper.

2. AR and MATH

Augmented Reality technology is an attractive prospect when trying to make Mathematics “real” to students that, more often than not, find only “intangible abstractions” in their concepts. This being, by default, a natural part of this science. The idea of AR inclusion, undoubtedly, provides motivational elements with consequences of improvement in the interest of mathematical studies. However, the goal proposed with this project is focused on a didactic intention of offering useful and applicable knowledge to students, in tandem with a new sense of accessibility from a cognitive point of view. In this sense, the present work involves the creation of a prototype that contemplates the development of cognitive abilities, which will be used in the relation of algebraic, numeric and graphic representations of mathematical knowledge. In addition, a new way for the role of technology is pursued, giving to the technological resource’s the assignment as a mediating feature between user and mathematical knowledge. The purpose of this interaction is the favoring of a co-action with digital technology, in such a way that it functions as a “cognitive partner” that contributes to reflect on learning, promotes visualization and, with it, Mathematics comprehension. The latter according to current theoretical frameworks in Mathematics Education [4, 7].

The construction of an Augmented Reality system implies the production of a virtual object superimposed in real time and place. It combines reality and 3-D-represented virtuality, promoting real time interaction with the object through its perception by the user. The virtual object shows information that the user does not detect directly through his bodily senses, and it is the transmission of this information that helps the user solve real world tasks; this is the use commonly given to AR in other areas. For the case described in this work, related to didactic applications, the impact of AR technologies is expected to be central in the activation of cognitive
processes in users, enabling a development of visualization skills and, with them, a spatial visualization ability. Toward this goal, the interaction between the student and the AR object must allow the evocation of processes that, in space and time, are related to the process of cognitive conception of a tridimensional object. When planning the way to collaborate in the project, the conceptualization of a complete product (prototype) was considered vital before beginning. With this, there came a necessity to gather content in the Mathematics curriculum in which the development of transversal competencies, not explicit in conventional syllabus, could be considered. The product should incite changes that will lead to an innovation in the learning of Mathematics that extends over the use of emerging technology to teach the same conventional content. The contribution of technology must directly affect the forms of interaction with mathematical knowledge, bringing new opportunities to learning. The challenge is towards a visual and tangible MATH. Figure 1 shows the application interface and a submenu of it.

![Application interface and submenu](image)

Fig. 1. (a) Application interface; (b) submenu of the application corresponding to the sine shape.

Analyzing mathematical content, decision was made to consider as themes: 2D to 3D, solids of revolution, and surfaces in space. These themes could be identified within courses of Mathematics I, II and III, but they are together in the AR application because this way 3D visualization is conceived as a whole learning object. The named project supported by Novus for one year, will cover the 2D to 3D theme.

3. AR: A promoter of visualization in the learning of Calculus

Once the themes that could be supported through AR were decided to be considered for the development of the ability of spatial visualization, the conceptualization of the prototype led to the discussion about the design of the product. The inclusion of video into the product was an important feature that will allow the explanation of some aspects about the way the animation will take place, supporting the learning process. Also the mathematical expressions should appear as part of the design to allow the identification of the graphical effect associated with the way a parameter affected the expression. Several ideas allowed finally to get into work trying to materialize the project that will lead to innovation.

The project was accepted with the next goals:

- Analysis of transversal content in conventional Calculus courses that is susceptible to a new AR base.
- Identification of a platform of technological and human resources where this may be produced.
- Development of AR in a didactic design prototype.
- Conducting of a pilot experience student trial for the description of this first prototype handling.

Getting to work it was understood that periodic sessions should be done, and there it become evident that in order to achieve the goals, there were two kinds of tasks that must take place in an organized way. In some
place there were academic aspects that led to the way things must be done, and in some place the technical concerns were the ones that needed to take the leadership in the work. The integration of the two kind of tasks naturally lead to focus on a parallel way of working in order to achieve a common goal. There the emergence of a transdisciplinary team was taking place.

The work plan considers a sequence of the activity of those two teams: technical and academic. To organize the development of the project, the next activities were considered:

- Conceptualization of the educational product
- Virtualization, Textures and Animation
- Augmented Reality Generation
- Academic validation of the product
- Graphical User Interface creation
- Technical validation of the product
- Educational research
- Analysis and reports

As a final product of the project, we have the didactic prototype with the corresponding Math content: from 2D to 3D visualization, as agreed for the first year of Novus 2012 support. The product already has the structure to load and integrate the content of Solids of Revolution and Surfaces in Space as soon as those modules are ready.

4. The AR Application

The first part of the application studies the transition from a 2D curve to a 3D surface; through ‘accumulation in time-space’ of different curves. It starts with a known pattern: a parabolic, sine or circular. The curves will be shaped by the graphical effect that corresponds to the presence of the parameter k in the algebraic expression. Different forms and graphical effects are included when considering the mathematical functions listed in Table 1.

<table>
<thead>
<tr>
<th>Graphical effect</th>
<th>Parabola</th>
<th>Sine</th>
<th>Circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>( y = x^2 )</td>
<td>( y = \sin x )</td>
<td>( y = \sqrt{1 - x^2} )</td>
</tr>
<tr>
<td>Vertical scroll</td>
<td>( y = x^2 + k )</td>
<td>( y = \sin x + k )</td>
<td>( y = \sqrt{1 - x^2 + k} )</td>
</tr>
<tr>
<td>Horizontal scroll</td>
<td>( y = (x + k)^2 )</td>
<td>( y = \sin (x + k) )</td>
<td>( y = \sqrt{1 - (x + k)^2} )</td>
</tr>
<tr>
<td>Contraction</td>
<td>( y = kx^2 )</td>
<td>( y = k \sin x )</td>
<td>( y = k\sqrt{1 - x^2} )</td>
</tr>
<tr>
<td>Expansion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Those images are originally in a 2D scenario and an animation occurs with the graphical effect, but still on the plane. The scenario changes to 3D when the parameter is acting and at the same time a copy of each of the successive curves is placed in a parallel plane situated so near the original one in such a way that a surface begins to take its own shape. With the effect of the parameter (k) and simultaneously the “motion of time” through the successive copies in parallel planes, the 3D visualization takes place. See Figure 2.
Fig. 2. Interactive 3D visualizations are displayed in the application, (a) shows a pale yellow plane and (b) shows a surface generated.

An important feature of the didactic design consists of an additional mark where a pale yellow plane appears with the intention that students could use it to interact with the previous AR surface generated (see Figure 2). This promotes the cognitive activity that benefits the spatial perception of the 3D object, like ‘cutting the surface’; something that in reality we could not do.

5. Educational research.

By February 2013 the technical validation of the prototype was done, and time was right to think on the design of an experience with students. The interest is to get in-sight, to discover, to interpret what is going on when students get to know the didactic prototype. Therefore the decision is to focus on qualitative case study.

In terms of [6] a qualitative case study is “an in-depth description and analysis of a bounded system”, in terms of [3], “case study research is a qualitative approach in which the investigator explores a bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple sources of information (e.g., observations, interviews, audiovisual material, and documents and reports), and reports a case description and case-based themes”.

The case is bounded by 30 students invited to participate in this study, all of them coursed Mathematics for Engineering I with Dr. Patricia Salinas. Each student was personally invited after a previous selection that considered their performance in some activities that involved visual aspects; limited, regular and good performance were included. When invited, students were told that it was just to obtain feedback for the Augmented Reality application for Mathematics. The study required them to spend a little of their non-class time in order to record a video showing the student-application reaction in pairs.

Each pair was selected according to the performance of each of its students, seeking diversity in each pair. After using the application, each student was asked to fill an open survey which featured 6 questions about their perception and opinion of different aspects of the AR application; such as availability and usability of the prototype in an electronic tablet, and if it was useful in order to develop any mathematical ability or if it provided knowledge that could be useful throughout their career.

This way, the case study involves an intrinsic interest from the researchers’ point of view: the experience of the 30 students as final users of the RA application. Studying the educational innovation using this kind of resource could bring insight about the new ways Mathematics education could experience. The descriptive and heuristic approach of the qualitative case study should illuminate this phenomenon.

Methods for data collection consist on the video sessions and a survey designed in order to compare the information obtained in both the video analysis and the survey analysis.
Students were given an Apple iPad with the application already installed, and two acrylic gadgets with the
marks used in the AR application. After finishing experimenting with the prototype, they answered the survey
individually. It is important to mention that there were no directions guiding the students, for they had to use
the materials provided intuitively. 15 pairs were recorded and 30 surveys were answered. They were analyzed
and the report is described below.

6. Analysis of videos and survey

Pairs of students were selected according to their grades, and aiming to get diversity of students in each pair.
Students with higher grades are more likely to visualize and identify mathematical 2-D graphs, so diversity was
a relevant matter. No other information was given to the students but what the goal of the activity was: to
recognize how intuitive the interface really is.

The analysis of these videos revealed the way in which students worked with the didactic prototype. In
general terms, students’ experience with the prototype was pretty intuitive.

Students interacted with both the tablet and the acrylic gadgets without any previous experience or getting
directions. Different approaches to proceed were observed; some students watched the video in order to get an
idea of what to expect, while some others started pressing buttons in order to understand its functions.

After looking at the 3D animation, some students even looked behind the tablet to confirm that there was no
physical object there. This observation suggests that 3D generated forms are more likely to invite the user to try
and physically manipulate them. See Figure 3.

A frequent reaction found in the students was that of reorienting perspective of the view with the acrylic
gadget to explore the form of the 3D object generated. Students were freely manipulating each element in the
application in order to get different perspectives of the model created by the ‘successive swept’ of a curve in
space. This phenomenon is possible due to the sensation of being able to manipulate a physical object, which is
created by the application. See Figure 4.
The little acrylic mark (which shows a yellowish, 3D plane) was used by the students in different ways. Some used it to represent time, as the generation of the model was followed synchronically to show how the model was formed. The most common use of this mark was the one conceived in its design: to act as a ‘blade’ to ‘cut’ the model, so students observed different curves in space that gave information about the function itself. This reaction is intended, so students can cognitively generate a perception of the surface as a consequence of the intersection of the mark-generated plane and the 3D model.

While watching the animation, students communicated in different ways. Some hand gestures were used as a way to explain how the model had been constructed, which lead to meaningful discussions between teammates. See Figure 5.

Watching the students’ actions, useful feedback about the application was collected. When they didn’t play the video included with the explanation and pressed the buttons instead, the algebraic expression was not shown accordingly to the model generated onscreen. This bug is now known and will be fixed in order to give students the real-time mathematical expression of what they’re seeing, thus they can relate a graphical function to a certain expression with given parameters.

The study also showed that the main ability that a user can get from the application is the understanding of time and space through visualization. The following is a quote from one of the students.
“It allows me to understand the behavior of a graph outside a paper sheet, and that its ‘change’ occurs in different dimensions. There’s something else going on in the front or in the back of the function.”

Other students mention the link between algebraic expressions and their functions.

The availability of the application in an electronic tablet was well accepted throughout the students, because of both its portability and attractive design.

Some students recognize that a visual aider helps them to easily understand the content, while others say it is an attractive way to learn math (which has been considered tedious since ages past).

In general terms, students considered that the application design was practical, simple and attractive. It is a friendly app, and the video is a nice addition, for “it looks like we’re chatting with someone about these topics”. They also mention that it is not hard to get any angle using a tablet, nor is hard to understand even without directions.

Nevertheless, they consider the acrylic marks need a tweak in order to be easier to use them along with the tablet. It is important to mention that while a student had the tablet, the other used the marks to interact with the model.

Here are some other comments about the usefulness of the app in their professional careers:

- “For the creation of animations or videogames, it could help me to view it from different angles and perspectives” (Computer Science)
- “In my career there are lots of analog waves and such, this application would be useful to view them as 3D waves.” (Sound Engineering & Production)
- “I can see how some biologic agents act.” (Food Industries)
- “My career always looks for innovation, and this is a very creative and a great way to learn math nowadays. Creative people are pretty adaptable and are always looking to go out their comfort-zone. Viewing math as AR is a great way to learn to let go off from pencil and paper.” (Industrial Systems)
- “It is always useful to observe how exponential phenomenon works, for it is vastly explored in my financial career” (Economy).

7. Final Reflection

A TEAM has emerged at Tecnológico de Monterrey. Today, Tecnología Educativa para el Aprendizaje de las Matemáticas, combines the use of emerging technologies and educational methods in order to achieve a better Math learning.

As TEAM there is the assumption of the challenge in creating a multi-disciplinary team that can lead an innovation process in our campus. Talking about innovation implies to assume a brave attitude towards whatever could come. Contact toward new discoveries and technology development is required in order to keep innovating.

The decision to work in different projects, applying to different Call for, and to look forward considering educational research, is the main thought that drives TEAM. The lack of human resources for development and research in each of the projects is an important matter; so far there is no stable solution. Support from immediate authorities has been key to the achievements so far.

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References